UCL DEPARTMENT OF MATHEMATICS

Book of abstracts for

Inverse problems and applications: a workshop in memory of Slava Kurylev

Department of Mathematics
University College London

18–19 April 2024

Organisers:
Giovanni Bracchi (UCL)
Matteo Capoferri (Heriot-Watt)
Dmitri Vassiliev (UCL)

Thursday 18 April 2024

10:00–11:00 **Bill Lionheart** (Manchester)
Anisotropy in diffraction tomography and radar

Many of the anisotropic phenomena that arise in inverse problems involve a second rank symmetric tensor as a material property to be imaged. We introduce two problems where higher rank symmetric tensors play this role in the form of spherical harmonic expansions of functions on sphere bundles.

As examples we consider problems in diffraction tomography and multistatic travel time imaging (eg radar), how these arise, what existing theory we can apply and some unsolved problems.

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11:00–12:00 **Matti Lassas** (Helsinki)
Geometric methods for manifold learning — A journey from applications to pure mathematics

We consider the invariant manifold learning (that is, the geometric Whitney problem) on how a Riemannian manifold can be constructed to approximate a given discrete metric space. This problem is closely related to invariant manifold learning, where a Riemannian manifold (M, g) needs to be approximately constructed from the noisy distances $d(X_j, X_k) + \eta_{jk}$ of points $X_1, X_2, \ldots, X_N$, sampled from the manifold M. Here, $d(X_j, X_k)$ are the distance of the points $X_j, X_k \in M$ and η_{jk} are either deterministic or random measurement errors. To study this problem we consider also learning of submanifolds of the high dimensional Euclidean spaces.

We also consider applications of the results in inverse problems encountered in medical and seismic imaging. In these problems, an unknown wave speed in a domain needs to be determined from indirect measurements. In geometric terms, this corresponds to the reconstruction of the Riemannian metric associated with the wave velocity from the wave kernel (or the heat kernel) measured in a subset of the domain.

The presented results have been done in collaboration with C. Fefferman, S. Ivanov, Y. Kurylev, J. Lu and H. Narayanan.

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14:00–15:00 **Daniel Lesnic** (Leeds)
Identification of the blood perfusion coefficient in a thermal-wave model of heat transfer in biological tissues

Many materials in nature possess properties that are unknown and difficult to measure directly. In such a situation, inverse modelling offers a viable alternative where one is trying to infer those unknown properties from appropriate measurements of

the main dependent variable(s) governing the physical process under investigation. Our investigation is driven by the fact that knowledge of the properties of biological tissues is essential in monitoring any abnormalities that may be forming and have a major impact on organs malfunctioning. Therefore, these disorders must be detected and treated early to save lives and improve the general health. Within the framework of thermal therapies, e.g. hyperthermia or cryoablation, the knowledge of the tissue temperature and of the blood perfusion rate are of utmost importance. Motivated by such a significant biomedical application, this study investigates the unique and stable reconstruction of the space-dependent (heterogeneous) blood perfusion coefficient in the thermal-wave hyperbolic model of bio-heat transfer from final-time temperature or boundary temperature data.

Joint work with M. Alosaimi and Y.-L. Fang. The support of the EPSRC grant EP/W000873/1 on "Transient Tomography for Defect Detection" is acknowledged.

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15:00–16:00 **Simon Arridge** (UCL)
Tomography with Sound and Light

Several different techniques exist for indirectly recovering the optical absorption and/or scattering coefficients of biological objects, and from there to inferring concentrations of chromophores of interest, from observations of transmitted and reflected light at multiple wavelengths; these include diffuse optical tomography, fluorescence optical tomography, and bioluminescence tomography. These modalities exhibit a tradeoff between greater contrast against lower resolution due to increased scattering. Acoustic waves also have a long tradition in imaging with both qualitative and quantitative interpretations. These concepts are combined in photo-acoustic tomography (PAT) which generates contrast with optical photons and develops resolution using ultrasound. In this talk I review some recent progress in these areas including the acceleration of PAT using Compressed Sensing and Machine Learning techniques

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16:30–17:30 **Katya Krupchyk** (UC Irvine)
Inverse problems for elliptic operators

We discuss recent progress on inverse problems for elliptic operators in the setting of Riemannian manifolds. The first part of the talk concerns the fractional anisotropic Calderon problem on closed Riemannian manifolds. We demonstrate that the knowledge of the local source-to-solution map for the fractional Laplacian, given on an arbitrary small open nonempty a priori known subset of a smooth closed Riemannian manifold, determines the Riemannian manifold up to an isometry. This can be viewed as a nonlocal analog of the anisotropic Calderon problem in the setting of closed Riemannian manifolds, which remains wide open in dimensions three and higher. The second part of the talk deals with inverse boundary problems for

semilinear Schrodinger equations on smooth compact Riemannian manifolds with boundary, at a large fixed frequency. We demonstrate that certain classes of cubic nonlinearities are uniquely determined from the knowledge of the nonlinear Dirichlet-to-Neumann map at a large fixed frequency on quite general Riemannian manifolds. In particular, in contrast to the previous results available, here the manifolds need not satisfy any product structure, may have trapped geodesics, and the geodesic ray transform need not be injective. Only a mild assumption about the geometry of intersecting geodesics is required.

The talk is based on joint work with Ali Feizmohammadi, Tuhin Ghosh, and Gunther Uhlmann, as well as on joint work with Shiqi Ma, Suman Kumar Sahoo, Mikko Salo, and Simon St-Amant.

Friday 19 April 2024

10:00–11:00 **Lauri Oksanen** (Helsinki)
Inverse problem for Yang-Mills-Higgs fields

We show that the Yang-Mills potential and Higgs field are uniquely determined (up to the natural gauge) from source-to-solution type data associated with the classical Yang-Mills-Higgs equations in the Minkowski space. We impose natural non-degeneracy conditions on the representation for the Higgs field and on the Lie algebra of the structure group which are satisfied for the case of the Standard Model. Our approach exploits non-linear interaction of waves to recover a broken non-abelian light ray transform of the Yang-Mills field and an integral transform of the Higgs field. The talk is based on joint work with Chen Xi, Matti Lassas, and Gabriel Paternain.

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11:00–12:00 **Alberto Ruiz Gonzalez** (Madrid)
Homogenization of elliptic equation in parallelizable manifolds

We introduce the two scale homogenization of elliptic equations in parallelizable and compact Riemannian manifold withn a choce od the parallelization as a bais of cells to take the limit and and describe the homogenizazed equations relating the metric and the frame. Even in the Euclidean case it allows to move the cells from point to point. Based on a work with Luis Guijarro, Daniel Faraco and Slava Kurilev.

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14:00–15:00 **Allan Greenleaf** (Rochester)
Heisenberg metamaterials

In the late 2000s-early 2010s, Slava, Henrik Kettunen, Matti Lassas, Gunther Uhlmann and I wrote a paper, which finally appeared as Superdimensional metamaterial resonators, SIAM Jour. Appl. Math. 78 (2018), no. 1, 437-456. We showed how structured composites (metamaterials) could be designed so they support wave propagation mimicking that associated with Grushin-type operators and (with many fewer details) more general sub-Riemannian geometries. This gave rise to several anomalous behaviors which we grouped together and labeled as 'superdimensional'. In fact, there was originally supposed to be a second paper, describing and analyzing superdimensional metamaterials based on the sub-Laplacian on the 3D Heisenberg group. Although this material is perhaps somewhat dated, I would like to present it in honor of Slava, since I think it was work that he was very proud of.