UCL DEPARTMENT OF MATHEMATICS

Book of abstracts for the

Workshop on Microlocal Analysis and PDEs

Department of Mathematics
University College London

20–22 July 2022

Organisers:
Matteo Capoferri (Cardiff)
Claudia Garetto (QMUL)
Dmitri Vassiliev (UCL)

Wednesday 20 July 2022

10:00–10:50 **Michael Ruzhansky** (Ghent)
Subelliptic pseudo-differential calculus on compact Lie groups

In this talk we will review recent advances in the development of the subelliptic pseudo-differential calculus (joint with with D. Cardona), bridging the gap with the already developed elliptic calculus on compact Lie groups (joint work with V. Turunen) and the subelliptic calculus for graded Lie groups (joint with with V. Fischer).

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11:00–11:50 **Sandro Coriasco** (Torino)
Microlocal analysis of stochastic partial differential equations

I will present various results, illustrating the employment of techniques coming from microlocal analysis to the study of different classes of linear and semilinear stochastic partial differential equations. These allow to prove existence and uniqueness of solutions, in suitably chosen functional spaces, under conditions on the involved noise. This is joint work with A. Ascanelli, A. Abdeljawad, and A. Süß, and with S. Pilipović and D. Seleši.

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12:00–12:25 **Ngoc Nhi Nguyen** (Paris Saclay)
Fermionic semiclassical L^p estimates

Spectral properties of Schrödinger operators are studied a lot in mathematical physics. They can give the description of trapped fermionic particles. This presentation will focus on the non-interacting case. I will explain why it is relevant to estimate L^p bounds of orthonormal families of eigenfuntions at the semiclassical regime and then, give the main ideas of the proof.

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14:00–14:50 **Jeffrey Galkowski** (UCL)
Classical wave methods and modern gauge transforms: spectral asymptotics in the one dimensional case
The question of high energy asymptotics for the kernel of the spectral projector of the Laplacian in the context of compact manifolds is one of the most well studied areas of spectral theory since the early 1900s. In this talk, we discuss the analogous question for Schrödinger operators on the real line: What are the asymptotics for the spectral projector of a Schrödinger operator on $\mathbb{R}$? By combining the classical wave method, originally introduced by Levitan in the 1950s, with the periodic gauge transform technique, we are able to show that when the potential is bounded with all derivatives this kernel, known as the local density of states, has a full asymptotic expansion in

powers of the spectral parameter. This proves a conjecture of Parnovski–Shterenberg in the one dimensional case.

Based on joint work with Leonid Parnovski and Roman Shterenberg

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15:00–15:30 **Katrina Morgan** (Northwestern)
Wave propagation on rotating cosmic string backgrounds – Part I

A rotating cosmic string spacetime has a singularity along a timelike curve corresponding to a one-dimensional source of angular momentum. Such spacetimes are not globally hyperbolic: there exist closed timelike curves near the "string". Nonetheless, we (Katrina Morgan and Jared Wunsch) have recently shown that forward solutions to the wave equation (in an appropriate microlocal sense) do exist.

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16:00–16:50 **Serena Federico** (Bologna)
On some variable coefficient NLS equations on the torus

In this talk we shall investigate the local well-posedness of the initial value problem (IVP) for some nonlinear Schrödinger (NLS) equations with variable coefficients on the torus. We will first show that suitable Strichartz estimates are valid in the toroidal setting for the operators under consideration. Next, as a consequence of the aforementioned inequalities, we will get sharp local well-posedness results for the associated nonlinear IVP.

Thursday 20 July 2022

10:00–10:50 **Alberto Parmeggiani** (Bologna)
On Kohn's sums of squares of complex vector fields

I will start by reviewing some hypoellipticity properties of operators of the kind sums of squares of complex vector fields, and then give a generalization of Kohn's subellipticity result.

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11:00–11:50 **Jared Wunsch** (Northwestern)
Wave propagation on rotating cosmic string backgrounds – Part II

A rotating cosmic string spacetime has a singularity along a timelike curve corresponding to a one-dimensional source of angular momentum. Such spacetimes are not globally hyperbolic: there exist closed timelike curves near the "string". Nonetheless, we (Katrina Morgan and Jared Wunsch) have recently shown that forward solutions to the wave equation (in an appropriate microlocal sense) do exist.

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12:00–12:25 **Yan-Long Fang** (Leeds)
A mathematical analysis of Casimir interactions

In this talk, I will briefly introduce three methods of calculating Casimir forces. The first one is due to Hendrik Casimir himself and it is known as the zeta regularisation of vacuum energy. The second one is due to Evgeny Lifshitz, which is based on a quantisation of stress energy tensor. The third one due to various groups of physicists and it is based on a Fredholm determinant of some boundary layer operators. It has been an open question whether all three methods are equivalent. My recent work with Alexander Strohmaier gives an affirmative answer to the problem. The key object object is a unique linear combination of various Laplace operators. It is called the relative Laplace operator and enjoys a Birman–Krein formula for a bigger class of functions. Moreover, it has a nice Hadamard variation formula for its resolvent, which builds a bridge between the second and the third methods of calculating Casimir forces. Applying heat kernel method to the relative Laplace operator, one could achieve the equivalence between the first and the second method. Finally, I will explain how one could use Duistermaat–Guillemin technique to study the relative wave trace invariance and it will show that the third method is more computationally efficient than the other two.

This talk is based on a joint work with Alexander Strohmaier.

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14:00–14:50 **Julie Rowlett** (Chalmers)
The mathematics of 'hearing the shape of a drum'

Have you heard the question "Can one hear the shape of a drum?" Do you know the answer? In 1966, M. Kac's article of the same title popularized the inverse isospectral problem for planar domains. Twenty-six years later, Gordon, Webb, and Wolpert demonstrated the answer, but many naturally related problems remain open today. We will discuss old and new results inspired by "hearing the shape of a drum."

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15:00–15:30 **Joey Zou** (UC Santa Cruz)
Microlocal Methods for The Elastic Travel Time Tomography Problem for Transversely Isotropic Media

I will discuss the travel time tomography problem for elastic media in the transversely isotropic setting. The mathematical study of this problem relates to X-ray tomography and boundary rigidity problems studied by de Hoop, Stefanov, Uhlmann, Vasy, et al., which reduce the inverse problems to the microlocal analysis of certain operators obtained from a pseudolinearization argument. In the previous works, the authors made strong use of the scattering pseudodifferential calculus, particularly using the inversion theory of elliptic scattering operators. However, in the current setting the analysis is more subtle, as the operators obtained are somewhat degenerate (they resemble parabolic operators in a particular sense, rather than elliptic operators in previous works). In this talk, I will explain the pseudolinearization argument and the qualitative features of the operators obtained, as well as the analysis required to accommodate the slightly more degenerate operators.

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Public Lecture

16:30–17:30 **Carola-Bibiane Schönlieb** (Cambridge)
Mathematical imaging: From geometric PDEs and variational modelling to deep learning for images

Images are a rich source of beautiful mathematical formalism and analysis. Associated mathematical problems arise in functional and non-smooth analysis, the theory and numerical analysis of nonlinear partial differential equations, inverse problems, harmonic, stochastic and statistical analysis, and optimisation.

In this talk we will learn about some of these mathematical problems, about variational models and PDEs for image analysis and inverse imaging problems as well as recent advances where such mathematical models are complemented and replaced by deep neural networks.

The talk is furnished with applications to art restoration, forest conservation and cancer research.

Friday 22 July 2022

10:00–10:50 **Maciej Zworski** (UC Berkeley)
Internal waves in 2D aquaria and homeomorphisms of the circle

The connections between the formation of internal waves in fluids, spectral theory, and homeomorphisms of the circle were investigated by oceanographers in the 90s and resulted in novel experimental observations (Leo Maas et al, 1997). The specific homeomorphism is given by a "chess billiard" and has been considered by many authors (Fritz John 1941, Vladimir Arnold 1957, Jim Ralston 1973...). The relation between the nonlinear dynamics of this homeomorphism and linearized internal waves provides a striking example of classical/quantum correspondence (in a classical and surprising setting of fluids!). I will illustrate the results with numerical and experimental examples and explain how classical concepts such as rotation numbers of homeomorphisms (introduced by Henri Poincare) are related to solutions of the Poincare evolution problem (so named by Elie Cartan). The talk is based on joint work with Semyon Dyatlov and Jian Wang.

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11:00–11:50 **Michał Wrochna** (Cergy Paris)
Lorentzian spectral zeta functions

The spectral theory of the Laplace–Beltrami operator on Riemannian manifolds is known to be intimately related to geometric invariants. These relationships have inspired many developments in relativistic physics, but a priori they do only apply to the case of Euclidean signature. In contrast, the physical setting of Lorentzian manifolds has remained problematic for very fundamental reasons.

In this talk I will present results that demonstrate that there is a well-posed Lorentzian spectral theory nevertheless, and it is related to Lorentzian geometry in a way that resembles results known so far only in Euclidean signature. In particular, in a joint work with Nguyen Viet Dang (Sorbonne Université), we show that the scalar curvature can be obtained as the pole of a spectral zeta function density. Furthermore, we equate the residue to a Pollicott–Ruelle resonance for a scaling dynamics and interpret the construction as a generalized Guillermou–Wodzicki residue density.

The primary consequence is that gravity can be obtained from a spectral action. Furthermore, that action arises as a scaling anomaly.

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12:00–12:25 **Andrea Sartori** (Tel Aviv)
The expected nodal volume of random non-Gaussian band limited functions.

Band limited functions are linear combinations of Laplace eigenfunctions on any smooth manifold. When their eigenvalues come from a small energy window (short

band), we expect the behavior of band limited functions to model the behavior of a single eigenfunction. In this talk, I will explain how this can be used to investigate the nodal volume, that is volume of the zero set of Laplace eigenfunctions. In particular, I will prove that the expected nodal volume of band limited functions, with random coefficients, is universal. Along the way, we will see how micro-local analysis techniques will be fundamental in the study of the covariance structure of the said functions.

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14:00–14:50 **Jonathan Eckhardt** (Loughborough)
On the inverse spectral transform for the conservative Camassa-Holm flow

The Camassa-Holm equation is a nonlinear partial differential equation that models unidirectional wave propagation on shallow water. I will show how this equation can be integrated by means of the inverse spectral transform method. The global conservative solutions obtained in this way form into a train of solitons (peakons) in the long-time limit.

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15:00–15:30 **Moritz Doll** (Bremen)
Heat Trace Asymptotics for the Generalized Harmonic Oscillator on Scattering Manifolds

On a scattering manifold, we consider a Schrödinger operator of the form $H = -\Delta + V(x)$, where the potential satisfies a growth condition that generalizes quadratic growth for Euclidean space. These types of operators were first investigated by Wunsch, who proved a relationship between singularities of the wave trace and a Hamiltonian flow. On the other hand, it is easy to see that the heat trace is smooth away from $t = 0$ and our goal is to calculate the asymptotic expansion of the heat trace as $t \to 0$. We follow the approach of Melrose by constructing a suitable space on which the integral kernel of the heat operator is smooth and then using the push-forward theorem to calculate the heat trace asymptotics. This is based on ongoing joint work with Daniel Grieser.

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